

Common Core Georgia Performance Standards CCGPS

Mathematics

Standards

CCGPS Analytic Geometry



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"Making Education Work for All Georgians"

CCGPS Analytic Geometry

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability.

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3) \cdot 3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

The Real Number System

N.RN

Extend the properties of exponents to rational exponents.

MCC9-12.N.RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{(1/3)}$ to be the cube root of 5 because we want $[5^{(1/3)}]^3 = 5^{[(1/3) \times 3]}$ to hold, so $[5^{(1/3)}]^3$ must equal 5.*

MCC9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

MCC9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations with complex numbers.

MCC9-12.N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

MCC9-12.N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

MCC9-12.N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find ~~moduli and~~ quotients of complex numbers.

Use complex numbers in polynomial identities and equations.

MCC9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p . Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1 + b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions

A.SSE

Interpret the structure of expressions

MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.*

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MCC9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .★

MCC9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

MCC9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.★

MCC9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.★

Arithmetic with Polynomials and Rational Expressions

A.APR

Perform arithmetic operations on polynomials

MCC9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations★

A.CED

Create equations that describe numbers or relationships

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.★

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .*★

Reasoning with Equations and Inequalities

A.REI

Solve equations and inequalities in one variable

MCC9-12.A.REI.4 Solve quadratic equations in one variable.

MCC9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

MCC9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

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Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

Solve systems of equations

MCC9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v ; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T .

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Interpreting Functions

F.IF

Interpret functions that arise in applications in terms of the context

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Analyze functions using different representations

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

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MCC9-12.F.IF.7a Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.*

MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MCC9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions

F-BF

Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models*

F.LE

Construct and compare linear, quadratic, and exponential models and solve problems

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

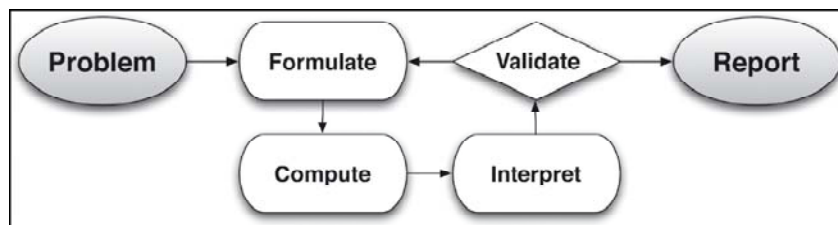
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or



statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

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In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO₂ over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).*

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Understand congruence in terms of rigid motions

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

MCC9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understand similarity in terms of similarity transformations

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

MCC9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

MCC9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

MCC9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Circles

G.C

Understand and apply theorems about circles

MCC9-12.G.C.1 Prove that all circles are similar.

MCC9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

MCC9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

MCC9-12.G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

MCC9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Translate between the geometric description and the equation for a conic section

MCC9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

MCC9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

Use coordinates to prove simple geometric theorems algebraically

MCC9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.*

Explain volume formulas and use them to solve problems

MCC9-12.G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

MCC9-12.G.GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

MCC9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Mathematics | High School—Statistics and Probability★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Interpreting Categorical and Quantitative Data

S.ID

Summarize, represent, and interpret data on two categorical and quantitative variables

MCC9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

MCC9-12.S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize ~~linear~~, quadratic, and ~~exponential~~ models.★

Understand independence and conditional probability and use them to interpret data

MCC9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

MCC9-12.S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

MCC9-12.S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

MCC9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

MCC9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MCC9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.★

MCC9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.★